IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

An Approach for Unreliability of Direct Methods-Optimal Solution of Transportation Problem

Pushpa Latha Mamidi[*1], M.S.R. Murthy[2]
[*1,2]Assistant Professor, of Mathematics, Dept. of Basic Science, Vishnu Institute of Technology, Bhimavaram, A.P., India
pushpamamidi@gmail.com

Abstract

The Transportation problem is a special class of Linear Programming Problem. It deals with shipping commodities from different sources to various destinations. The objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. In general, the transportation model can be extended to areas other than the direct transportation of a commodity, including among others, inventory control, Engineering & Technology, Management sciences, Employment scheduling. In this paper, I have tried to reveal that the proposed direct methods namely NMD Method, Exponential Approach for finding optimal solution of a Transportation problem do not present optimal solution at all times.

Keywords: Transportation Problem, VAM, VAM-MODI, NMD Method, Exponential Approach Method, Optimal Solution, reliable.

Introduction

A Transportation problem is one of the earliest and most important applications of linear programming problem. Description of a classical transportation problem can be given as follows. A certain amount of homogeneous commodity is available at number of sources and a fixed amount is required to meet the demand at each number of destinations. A balanced condition (i.e., total demand is equal to total supply) is assumed. Then finding an optimal schedule of shipment of the commodity with the satisfaction of demands at each destination is the main goal of the problem. In 1941 Hitchcock [1] developed the basic transportation problem along with the constructive method of solution and later in 1949 Koopmans [3] discussed the problem in detail. Again in 1951 Dantzig [2] formulated the transportation problem as linear programming problem and also provided the solution method. Now a day's transportation problem has become a standard application for industrial organizations having several manufacturing units, warehouses and distribution centers.

Among them some methods have been introduced which directly attain at the optimal solution namely NMD Method [6], Exponential Approach [7] etc. Also it can be said that those methods reveal optimal solution without disturbance of degeneracy condition. There requires least iterations to reach optimality, compared to the existing methods available in the literature. The degeneracy problem is also avoided by those methods. In Exponential Approach methods much easier heuristic approach is proposed for finding an optimal solution directly with lesser number of iterations and very easy computations. But from time to time there occur few evils that, the optimal solutions found by them are not actual. In this paper, I have presented that the proposed direct methods for finding optimal solution of a transportation problem do not reflect optimal solution continuously. Three numerical examples are provided to prove my claim. Also by the VAM-Modi process optimal solutions are showed to illustrate the comparison.

Feasible Solution (F.S)

A set of non negative allocation $x_{ij} \geq 0$ which satisfies the row and column restriction is known as Feasible Solution

Basic Feasible Solution (BFS)

A feasible solution to a m-origins and n-destination problem is said to be Basic Feasible Solution if the number of positive allocation are (m +

n -1), it is called Degenerate Basic Feasible Solution (DBFS) otherwise non-degenerate.

Optimal Solution

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

Balanced & Unbalanced Transportation Problem

A transportation problem is said to be balanced if the total supply from all sources equals the total demand in the destinations and is called unbalanced otherwise.

Thus for a balanced problem, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ *and*

For unbalanced problem, $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$

Optimality Test

Optimality test can be performed if "the number of allocation cells in an initial basic feasible solution = m+n–1" (No. of rows + No. of columns–1). Otherwise optimality test cannot be performed.

The object of optimality test is that, if we put an allocation in a vacant cell then whether the total transportation cost decreased.

Two methods for optimality test namely "Stepping Stone Method" and "MODI Method" are usually used whereas "MODI Method" is mostly used.

Modified Distribution Method (Modi Method) or U–V Method

This method follows the following steps:

Step– 1: Take the costs only at the cells where allocations are made. It is called cost matrix for allocated cells.

Step– 2: At the bottom of each column we put v_1, v_2, ……v_n and at the same time on the right of each row we put u_1, u_2,……u_m so that the sum of corresponding u's and v's in every allocated cell is equal to cell cost. Then by algebraic calculation, the values of each u's and v's are to be found out. It is called $u_i + v_j$ matrix for allocated cells.

Step - 3: The empty cells are filled up by the sum results of corresponding u's and v's.

Step-4: Subtract the above matrix's cells from the corresponding cells of original matrix. It is called cell evaluation matrix.

Step-5: If the above cell evaluation matrix contains only non-negative values then the basic feasible solution is optimal.

On the other hand, if any of the above cell evaluation matrix contains negative value, then the basic feasible solution is not optimal. Then for optimal solution the following iteration should be run:

Step-1: Select the most negative cell from the above cell evaluation matrix. If there have more than one equal cell, then any one can be chosen.

Step-2: Write the initial basic feasible solution. Give a tick (√) at the most negative entry cell. It is scaled identified cell.

Step-3: Trace or draw a path in this matrix consisting of a series of alternatively horizontal and vertical lines. The path begins and terminates in the identified cell. All corners of the path lie in the cells for which allocations have been made, the path may skip over any number of occupied or vacant cells.

Step-4: Mark the identified cells as positive and each occupied cell at the corners of the path alternatively negative, positive, negative and so on.

Step-5: Make a new allocation in the identified cell by entering the smallest allocation on the path that has been assigned a negative sign. Add and subtract this new allocation from the cells at the corners of the path, maintaining the row and column requirements. This causes one basic cell to become zero and other cells remain non-negative. The basic cell whose allocation has been made zero, leaves the solution.

VAM-MODI Process:

VAM-MODI process indicates, initial basic feasible solution is calculated by "Vogel's Approximation Method" first then optimality test can be checked out by "Modified Distributive Method".

NMD Method

Algorithm for solving Transportation problem

Step-1: Construct the Transportation matrix from given transportation problem.

Step-2: Select minimum odd cost from all cost in the matrix.

Step-3: Subtract selected least odd cost only from odd cost in matrix. Now there will be at least one zero and remaining all cost become even.

Step-4: Multiply by $\frac{1}{2}$ each column (i.e., $\frac{1}{2} C_{ij}$) or To get minimum cost 1 in any column, if possible by dividing cost in that column

Step-5: Again select minimum odd cost in the remaining column except zeros in the column

Step-6: Go to step 3. Now there will be at least one zero and remaining all cost will become even

Step-7: Repeat step 4 and 5, from remaining sources and destinations till (m+n–1) cells are allocated

Step- 8: Start the allocation from minimum of supply and demand. Allocate this minimum of supply/demand at the place of 0 first and then 1

Step-9: Finally total minimum cost is calculated as sum of the product of cost and corresponding allocated value of supply/demand

i.e., Total Cost = $\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$

ISSN: 2277-9655
Impact Factor: 1.852

Exponential Approach Method

Step-1: Construct the transportation model (Table) from the given transportation problem

Step-2: Subtract each row entries of the transportation table from the respective row minimum and then subtract each column entries of the transportation table from the respective column minimum, so that each row and column will have least one zero.

Step-3: Now there will be at lest one zero in each row and column in the reduced cost matrix. Select the first zero (row wise) occurring in the cost matrix. Count the total number of zeros excluding the selected one in the corresponding row and column. And then assign exponential penalties (sum of zeros in respective row and column). Repeat the procedure for all zeros in the matrix.

Step-4: Now choose a zero for which the minimum exponential penalty is assigned from step 3 and allocate the respective cell value with maximum possible amount.

If tie occurs for any cell in the penalty values then first check for the corresponding value in demand and supply, find its average value and assign the allocation for least average value. And if again tie occurs then check the corresponding value in the rows and column and select the minimum.

Step-5: After performing step 4 delete the row or column (where supply or demand becomes zero) for further calculation.

Step-6: Check whether the resultant matrix possesses at least one zero in each column and in each row. If not repeat step2, otherwise go to step7.

Step-7: Repeat step 3 to step 6 until and unless all the demands are satisfied and all the supplies are exhausted.

Step-8: For the allocated values calculate the optimal cost.

Numerical Examples

Example1: Consider the following cost minimizing transportation problem with 4 origins and 6 destinations

	D_1	D_2	D_3	D_4	D_5	D_6	**Supply**
O_1	9	12	9	6	9	10	**5**
O_2	7	3	7	7	5	5	**6**
O_3	6	5	9	11	3	11	**2**
O_4	6	8	11	2	2	10	**9**
Demand	**4**	**4**	**6**	**2**	**4**	**2**	**22**

Solution by VAM-MODI $x_{13} = 5$, $x_{22} = 3$, $x_{23} = 1$, $x_{26} = 2$, $x_{31} = 1$, $x_{32} = 1$, $x_{41} = 3$, $x_{44} = 2$, $x_{45} = 4$

Total Cost = (9*5) + (3*3) + (7*1) + (5*2) + (6*1) + (5*1) + (6*3) + (2*2) + (2*4) = 112

Solution by NMD Method $x_{13} = 5$, $x_{22} = 4$, $x_{26} = 2$, $x_{31} = 1$, $x_{33} =1$, $x_{41} = 3$, $x_{44} = 2$, $x_{45} = 4$

Total Cost = (9*5) + (3*4) + (5*2) + (6*1) + (9*1) + (6*3) + (2*2) + (2*4) = 112

Solution by Exponential Approach Method $x_{13} = 5$, $x_{22} = 4$, $x_{26} = 2$, $x_{31} = 2$, $x_{41} = 2$, $x_{43} = 1$, $x_{44} = 2$, $x_{45} = 4$

Total Cost = (9*5) + (3*4) + (5*2) + (6*2) + (6*2) + (11*1) + (2*2) + (2*4) = 114

Example2: Construct the following cost minimizing transportation problem with 4 origins and 3 destinations

	D_1	D_2	D_3	**Supply**
O_1	2	7	4	**5**
O_2	3	3	1	**8**
O_3	5	4	7	**7**
O_4	1	6	2	**14**
Demand	**7**	**9**	**18**	**34**

Solution by VAM-MODI $x_{11} = 5$, $x_{22} = 2$, $x_{23} = 6$, $x_{32} = 7$, $x_{41} = 2$, $x_{43} = 12$

Total Cost = (2*5) + (3*2) + (1*6) + (4*7) + (1*2) + (2*12) = 76

Solution by NMD Method $x_{12} = 2$, $x_{13} = 3$, $x_{23} = 8$, $x_{32}= 7$, $x_{41} = 7$, $x_{43} = 7$

Total Cost = (7*2) + (4*3) + (1*8) + (4*7) + (1*7) + (2*7) = 83

Solution by Exponential Approach Method $x_{11} = 5$, $x_{23} = 8$, $x_{32} = 7$, $x_{41} = 2$, $x_{42} = 2$, $x_{43} = 10$

Total Cost = (2*5) + (1*8) + (4*7) + (1*2) + (6*2) + (2*10) = 80

Example3: Consider the following cost minimizing transportation problem with 3 origins and 5 destinations

	D_1	D_2	D_3	D_4	D_5	**Supply**
O_1	4	1	2	4	4	**60**
O_2	2	3	2	2	3	**35**
O_3	3	5	2	4	4	**40**
Demand	**22**	**45**	**20**	**18**	**30**	**135**

Solution by VAM-MODI $x_{12}= 45$, $x_{15} = 15$, $x_{21} = 17$, $x_{24} = 18$, $x_{31} = 5$, $x_{33} = 20$, $x_{35} = 15$

Total Cost = (1*45) + (4*15) + (2*17) + (2*18) + (3*5) + (2*20) + (4*15) = 290

Solution by NMD Method $x_{12} = 45$, $x_{13} = 15$, $x_{21} = 17$,
$x_{24} = 18$, $x_{31} = 5$, $x_{33} = 5$, $x_{35} = 30$

Total Cost = (1*45) + (2*15) + (2*17) + (2*18) + (3*5) + (2*5) + (4*30) = 290

Solution by Exponential Approach Method $x_{12} = 45$, $x_{15} = 15$, $x_{21} = 2$, $x_{24} = 18$, $x_{25} = 15$, $x_{31} = 20$, $x_{33} = 20$
Total Cost = (1*45) + (4*15) + (2*2) + (2*18) + (3*15) + (3*20) + (2*20) =290

Conclusion

The results of NMD Method and Exponential Approach method create a doubt as the solution is optimal or not. In 5.1, the VAM-MODI process shows that the optimal solution is 112 and it is exact and NMD Method gives the same result. But the Exponential Approach gives the wrong result which is not optimal. In 5.2, the VAM-MODI process shows that the optimal solution is 76 whereas NMD Method and Exponential Approach Method give the wrong results which are not optimal. In 5.3, the VAM-MODI process displays that the optimal solution is 290 and at the same time NMD Method and Exponential Approach Method give the same optimal solution. By this one cannot decide whether the solution is optimal or not. So it cannot be wise to depend on the optimal solution found by them.

References

[1] *F.L. Hitchcock, "The distribution of a product from several sources to numerous localities", Journal of Mathematical Physics, Vol. 20, pp. 224-230, 2006*

[2] *G.B. Dantzig, Linear Programming and Extensions, Princeton University Press, Princeton, N J, 1963*

[3] *Koopams T.C., Optimum Utiliztion of Transportation System, Econometrica, Supplement vol. 17, 1949*

[4] *Mohammad Kamrul Hasan, "Direct Methods for Finding Optimal Solution of a Transportation Problem are not Always Reliable", International Refereed Journal of Engineering and Science ,Vol.1, pp. 46-5*

[5] *Abdul Quddoos, Shakeel Javaid, M.M. Khalid, "A New Method for Finding an Optimal Solution for Transportation Problems", International Journal on Computer Science and Engineering, Vol. 4 , No. 7, July 2012*

[6] *N.M. Deshmukh, "An Innovative Method For Solving Transportation Problem", International Journal of Physics and Mathematical Sciences, Vol. 2(3), pp. 86-91, 2012*

[7] *S. Ezhil Vannan, S. Rekha, "A New Method for Obtaining an Optimal Solution for Transportation Problems", International Journal of Engineering and Advanced Technology, Vol. 2, Issue-5, June 2013*

[8] *P. Pandian and D. Anuradha, "A New Approach for Solving Solid Transportation Problems," Applied Mathematical Sciences, Vol. 4, No. 72, 2010*

[9] *Kirca, O and A, Satir (1990), "A heuristic for obtaining an initial solution for the transportation problem", Journal of Operational Research Society, 41(9), pp. 865-871*

[10] *V.J. Sudhakar, N. Arunsankar, T. Karpagam, "A New Approach for Finding an Optimal Solution for Transportation Problems", European Journal of Scientific Research, Vol. 68, No.2, pp. 254-257, 2012*

[11] *Hamdy, T. Operations Research, An Introduction (6th Edn), Pearson Education Inc, 2002*

[12] *Taha, H.A., Operations Research – Introduction, Prentice Hall of In dia (PVT), New Delhi, 2005*

[13] *P. Pandian and G. Natarajan, "A New Method for Solving Bottleneck-Cost Transportation Problems", International Mathematical Forum, Vol.6, No. 10, pp. 451-460, 2010*

[14] *P. Pandian and G. Natarajan, "A new method for finding an optimal solution for Transportation Problems", International Journal of Math.Sci. & Engg. Appns.Vol. 4,pp- 59 - 65, 2010*